East Midlands Stochastic Analysis Seminar

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Organisers:

Zdzislaw Brzezniak (York), David Elworthy (Warwick), Chunrong Feng (Loughborough), Zhongmin Qian (Oxford), Huaizhong Zhao (Loughborough)

Department of Mathematical Sciences, Loughborough University
12 December 2019, Venue: SCH013

Talks

14:00-15:00: **Sergei Kuksin (Paris 7, France)**,
One-dimensional turbulence and the stochastic Burgers equation

15:00-15:30: Tea/Coffee

15:30-16:30: **Shizan Fang (Bourgogne, France)**
Vorticity, Helicity and intrinsic geometry for Navier-Stokes equations

16:30-17:30**: Björn Schmalfuss (Jena, Germany)**
Random Dynamical Systems generated by stochastic partial differential equations driven by fractional Brownian motion

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Sergei Kuksin (Paris 7, France),

Title: One-dimensional turbulence and the stochastic Burgers equation

Abstract: My talk is a review of the results on turbulence in the 1d viscous Burgers equation on a circle, obtained by myself and my former PhD students, Andrey Biryuk and Alexandre Boritchev; now the results are presented in a MS of my joint book with A.Boritchev with the same title as the talk. Namely, I will speak about the Burgers equation on a circle, perturbed by a random force which is smooth in x and white in time t, and explain that Sobolev norms of its solutions admit upper and lower estimates, which are asymptotically sharp as the viscosity goes to zero. This assertion allows to derive for solutions of the equation results, which are rigorous analogies of the main predictions of the Kolmogorov theory of turbulence. Namely, of the Kolmogorov laws for the short-scale increments of the turbulent vector-fields, and of the Kolmogorov-Obukhov law for the energy spectrum of turbulence (I will explain these notions). The results were non-rigorously obtained by physicists Aurell-Frisch-Lutsko-Vergassola in 1992 (and by Burgers in 1948, even more heuristically).

Shizan Fang (Bourgogne, France)

Title: Vorticity, Helicity and intrinsic geometry for Navier-Stokes equations

Abstract: Using the concept of rolling a Brownian motion with a drift on a Riemannian manifold, to a velocity, solution of Navier-Stokes equations, we will associate an affine metric connection so that associated vorticity, helicity can be linked through a time-dependent intrinsic Ricci tensor. The talk is based on a recent work with Zhongmin Qian.

Björn Schmalfuss (Jena, Germany)

Title: Random Dynamical Systems generated by stochastic partial differential equations driven by and fractional Brownian motion

Abstract: The generation of random dynamical systems by Ito equations with general diffusion coefficient is complicated, in the infinite dimensional it is an unsolved problem. The reason is that Ito integrals are defined only almost surely. But such a definition contradicts the definition of a random dynamical system. In particular often investigating the dynamics of spde it is restricted to very simple diffusion coefficients, namely additive noise and simple multiplicative noise.

New integration techniques (integration by fractional derivatives and rough path integration) provides pathwise stochastic integrals in particular for integrators given by the fractional Brownian motion. In this talk I will demonstrate how the how these (s)pde's generate a random dynamical system. In addition we will How we can show the existence of random attractors for non trivial diffusion coefficients when the Hurst parameter H of the fractional Brownian motion is in $(1/2,1)$ and local exponential stability when $H\in (1/3,1/2)$.